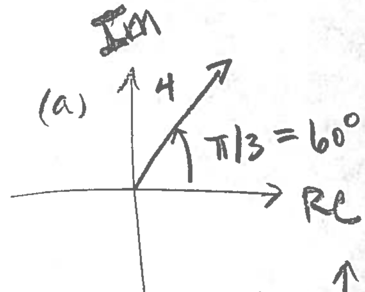
Ryan Chilton
ECE 311 HW #1

1.4) (a) $\tilde{z}_1 = 4e^{j\pi/3}$

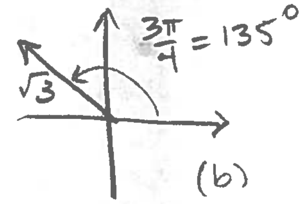
$$= 4\cos(\pi/3) + j4\sin(\pi/3)$$

$$\cong 2 + j3.464$$



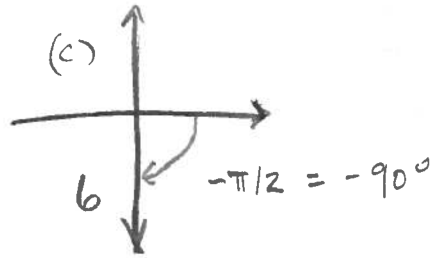
(b) $\tilde{z}_2 = \sqrt{3}\, e^{j3\pi/4}$

$$\cong -1.225 + 1.225j$$



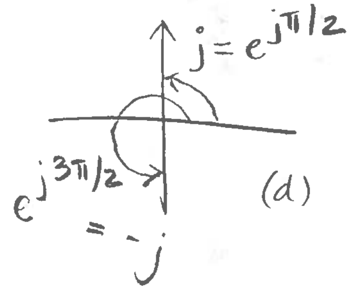
(c) $\tilde{z}_3 = 6e^{-j\pi/2}$

$$= -6j$$



(d) $\tilde{z}_4 = j^3 = (e^{j\pi/2})^3$

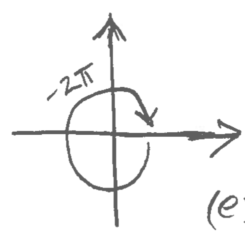
$$= e^{j3\pi/2} = -j$$



(OR $\tilde{z}_4 = j^3 = j^2 \cdot j = -1 \cdot j = -j$)

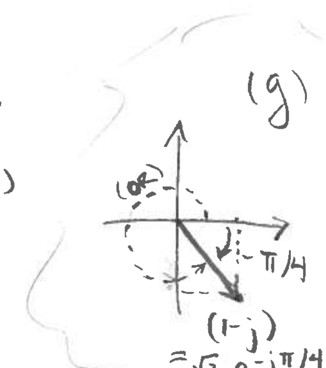
(e) $\tilde{z}_5 = j^{-4} = (e^{j\pi/2})^{-4}$

$$= e^{-j2\pi} = 1 \quad (!)$$



(f) $\tilde{z}_6 = (1-j)^3 = (1-j)(1-j)(1-j)$

$$= (1-j-j+j^2)(1-j) = (-2j)(1-j) = -2j + 2j^2 = -2-2j$$



(g) $\tilde{z}_7 = (1-j)^{1/2} = \left(\sqrt{2}\, e^{-j\pi/4}\right)^{1/2}$

- sqrt the magnitude
- half the angle

$$= \sqrt{\sqrt{2}}\, e^{-j\pi/8} \cong 1.099 - 0.455j$$

OR

$(1-j)^{1/2} = \sqrt{2}\, e^{+j\frac{7\pi}{4}}$

- sqrt the mag
- half the angle

$$\sqrt{\sqrt{2}}\, e^{+j\frac{7\pi}{8}} = -1.099 + 0.455j$$

(1.22) Use ~~tabel~~ table 1-5 ... I indicate which transform I used.

(a) $v(t) = 3\cos(\omega t - \pi/3) \longrightarrow \tilde{V} = 3e^{-j\pi/3}$ (LINE #2)

(b) $v(t) = 12\sin(\omega t + \pi/4) \longrightarrow \tilde{V} = 12e^{+j(\pi/4 - \pi/2)}$

$= 12e^{j(-\pi/4)}$ (LINE #6)

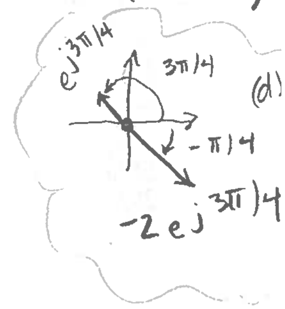
(c) $i(x,t) = 2e^{-3x}\sin(\omega t + \pi/6)$ — $\sin(\alpha) = \cos(\alpha - \pi/2)$

$= 2e^{-3x}\cos(\omega t + \pi/6 - \pi/2)$

$= 2e^{-3x}\cos(\omega t - \pi/3) \longrightarrow \tilde{I}(x) = 2e^{-3x}e^{-j\pi/3}$ (LINE #4)

(d) $i(t) = -2\cos(\omega t + 3\pi/4) \xrightarrow[\text{(LINE \#2)}]{} \tilde{I} = -2e^{j3\pi/4}$

$= 2e^{-j\pi/4}$



(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$

$\downarrow$ (LINE #6) $\qquad \downarrow$ (LINE #2)

$= 4e^{j(\pi/3 - \pi/2)} + 3e^{-j\pi/6}$

$= 4e^{-j\pi/6} + 3e^{-j\pi/6} = 7e^{-j\pi/6}$

(1.23) (a) $\tilde{V} = -5e^{j\pi/3}$ — Use $v(t) = \mathrm{Re}\{\tilde{V}e^{+j\omega t}\}$

$v(t) = \mathrm{Re}\{-5e^{j\pi/3}e^{j\omega t}\} = \mathrm{Re}\{-5e^{j(\omega t + \pi/3)}\}$

use eulers formula $e^{j\theta} = \cos\theta + j\sin\theta$

$= \mathrm{Re}\{-5\cos(\omega t + \pi/3) - 5j\sin(\omega t + \pi/3)\}$

$= -5\cos(\omega t + \pi/3)$

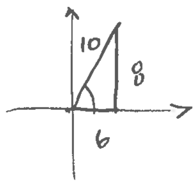
(b) $\tilde{V} = j6e^{-j\pi/4} = e^{j\pi/2} \cdot 6 \cdot e^{-j\pi/4}$

$= 6e^{+j\pi/4}$ ... (same procedure as (a) from here)

$\vdots$

$v(t) = 6\cos(\omega t + \pi/4)$

(c) $\tilde{I} = 6 + j8 = \left(\sqrt{6^2+8^2}\right) e^{+j\left(\operatorname{atan}(8/6)\right)}$

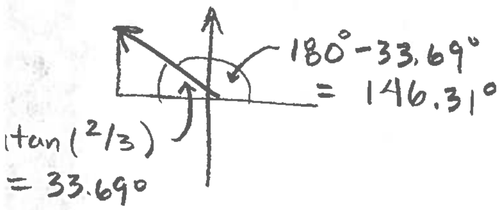


$= 10\, e^{+j\,0.923}$ ... same procedure as (a) from here

$\vdots$

$i(t) = 10 \cos(\omega t + 0.923)$

(d) $\tilde{I} = -3 + j2 \cong \sqrt{13}\, e^{j(146.31°)}$



$\vdots$ ... ″

$i(t) = \sqrt{13} \cos(\omega t + 146.31°)$

$= \sqrt{13} \cos(\omega t + 2.554 \text{ rad})$

(e) $\tilde{I} = j = e^{j\pi/2}$

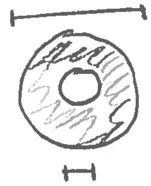
$\vdots$ ... ″

$i(t) = \cos(\omega t + \pi/2)$

(f) $\tilde{I} = 2 e^{+j\pi/6}$

$i(t) = 2 \cos(\omega t + \pi/6)$

(2.2) Coaxial line



$a = 0.25$ cm
$b = 0.5$ cm
$f = 1$ GHz $\rightarrow \omega = 2\pi \times 10^{9}$ $\frac{rad}{sec}$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \sqrt{\frac{\pi f \mu_c}{\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right)\cdot\frac{1}{2\pi}$$

$\mu_c = 4\pi \times 10^{-7}$ H/m
$\sigma_c = 5.8 \times 10^{7}$ S/m
$\leftarrow$ substitute

$= \cancel{4.95\ \Omega/m}$ forgot $\frac{1}{2\pi}$

$= 0.7888\ \Omega/m = 788$ mΩ/m

$$L' = \frac{\mu}{2\pi}\log_e(b/a) = 1.39 \times 10^{-7}\ H/m$$
$= 139$ nH/m

$$G' = \frac{2\pi\sigma}{\ln(b/a)}$$ $\leftarrow \sigma = 10^{-3}$ S/m

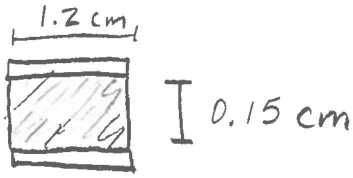
$= 0.0091$ S/m
9.1 mS/m

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)}$$

$\varepsilon = \underbrace{4.5}_{\varepsilon_r} \cdot \underbrace{(8.85 \times 10^{-12}\ F/m)}_{\varepsilon_0} = 3.98 \times 10^{-11}$ F/m

$$= \frac{2\pi \cdot 3.98 \times 10^{-11}\ F/m}{\log_e(0.5/0.25)} = 3.61 \times 10^{-10}\ F/m$$
$= 36.1$ pF/m

(2.3) Parallel Plate



$\mu_c = \mu = 4\pi \times 10^{-7}$ H/m

$\sigma_c = 5.8 \times 10^{7}$ S/m

$\varepsilon = 2.6\,\varepsilon_0 = 2.3 \times 10^{-11}$ F/m

$\sigma_{polystyrene} = 0$ $\quad f = 1\,\text{GHz}$

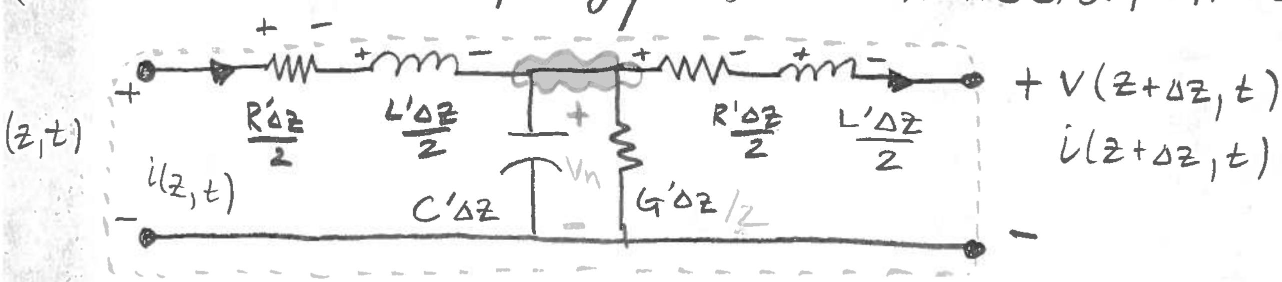
$$R' = \frac{2R_s}{w} = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = 1.375\ \Omega/\text{m}$$

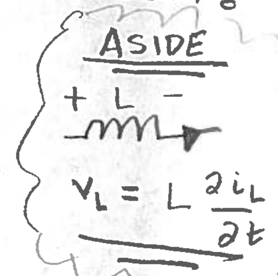
$$L' = \frac{\mu d}{w} = 1.57\ \text{H/m}$$

$$G' = \frac{\sigma w}{d} = 0\ \text{S/m}$$

$$C' = \frac{\varepsilon w}{d} = \frac{2.3\times 10^{-11}\ \text{F/m} \cdot 1.2\ \text{cm}}{0.15\ \text{cm}} = 1.84 \times 10^{-10}\ \text{F/m}$$

(2.4) Alternate topology of transmission line circuit.





THE EASY PART

Apply KCL on the dashed loop.. Σ drops = Σ rises

$$v(z,t) = v(z+\Delta z, t) + \frac{R'\Delta z}{2} i(z,t) + \frac{R'\Delta z}{2} i(z+\Delta z, t)$$
$$+ \frac{L'\Delta z}{2} \frac{\partial}{\partial t} i(z,t) + \frac{L'\Delta z}{2} \frac{\partial}{\partial t} i(z+\Delta z, t)$$

Rearrange, divide by Δz...

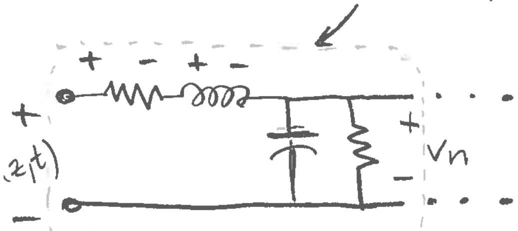
$$\frac{v(z+\Delta z, t) - v(z,t)}{\Delta z} = \frac{R'}{2} i(z,t) + \frac{R'}{2} i(z+\Delta z, t) \xrightarrow{\lim} i(z,t)$$
$$+ \frac{L'}{2} \frac{\partial}{\partial t} i(z,t) + \frac{L'}{2} \frac{\partial}{\partial t} i(z+\Delta z, t) \xrightarrow{\lim} i(z,t)$$

Take limit as $\Delta z \to 0$. Now $i(z+\Delta z, t)$ converges to $i(z,t)$
Those terms double up, canceling the 1/2

$$-\frac{\partial}{\partial z} v(z,t) = R' i(z,t) + L' \frac{\partial}{\partial t} i(z,t) \qquad \text{[eq. 2.14]}$$

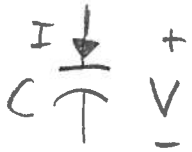
THE TRICKIER PART

Define the voltage at the middle node as $V_n(t)$
Use KCL on this loop to compute it in terms of $v(z,t)$



$$V_n(t) = v(z,t) - \frac{R'\Delta z}{2} i(z,t) - \frac{L'\Delta z}{2} \frac{\partial}{\partial t} i(z,t)$$

Another aside - capacitor ckt. relation ...



$$i_c = C \frac{\partial}{\partial t} v_c$$

Apply KCL at the $V_n(t)$ node

$$i(z+\Delta z,t) = i(z,t) - (C'\Delta z)\frac{\partial}{\partial t} V_n(t) - (G'\Delta z)\, V_n(t)$$

Back substitute the formula for $V_n(t)$. Rearrange.

$$i(z+\Delta z,t) - i(z,t) = -(G'\Delta z)\left[v(z,t) - \left(\frac{R'\Delta z}{2}\right) i(z,t) - \left(\frac{L'\Delta z}{2}\right)\frac{\partial}{\partial t} i(z,t)\right]$$
$$- (C'\Delta z)\left[\frac{\partial}{\partial t} v(z,t) - \left(\frac{R'\Delta z}{2}\right)\frac{\partial}{\partial t} i(z,t) - \left(\frac{L'\Delta z}{2}\right)\frac{\partial^2}{\partial t^2} i(z,t)\right]$$

Divide by Δz, and take $\lim \Delta z \to 0$

$$\lim_{\Delta z \to 0}\left(\frac{i(z+\Delta z,t) - i(z,t)}{\Delta z}\right) = \lim_{\Delta z \to 0}\left(-G'\left[v(z,t) - \cancel{\left(\frac{R'\Delta z}{2}\right)} i(z,t) - \cancel{\left(\frac{L'\Delta z}{2}\right)}\frac{\partial}{\partial t} i(z,t)\right] - C'\left[\frac{\partial}{\partial t} v(z,t) - \cancel{\left(\frac{R'\Delta z}{2}\right)}\frac{\partial}{\partial t} i(z,t) - \cancel{\left(\frac{L'\Delta z}{2}\right)}\frac{\partial^2}{\partial t^2} i(z,t)\right]\right)$$

Limit procedure makes a derivative on the Left Hand Side (LHS)
In the RHS, any term that gets multiplied by Δz shrinks to 0.

$$\frac{\partial}{\partial z} i(z,t) = -G' v(z,t) - C'\frac{\partial}{\partial t} v(z,t)$$

$$-\frac{\partial}{\partial z} i(z,t) = G' v(z,t) + C'\frac{\partial}{\partial t} v(z,t) \qquad [\text{eq. } 2.16]$$