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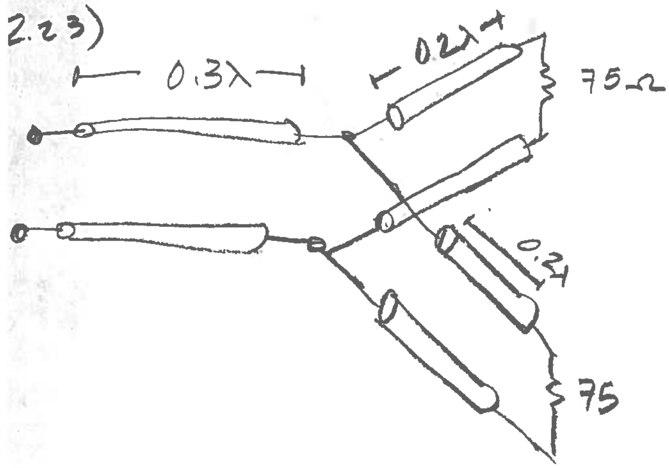
ECE 311 - HW#3 Solutions

2.23)

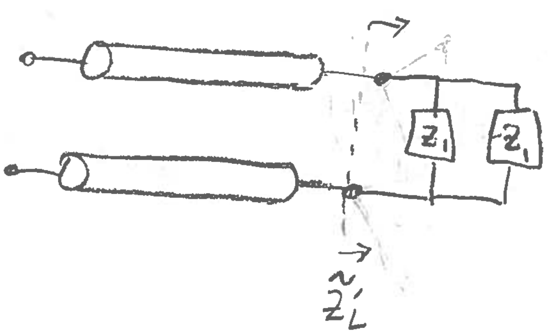


(a) Find the input impedance of one antenna at the Y-junction:

$$\tilde{Z}_1 = Z_0 \cdot \frac{\tilde{Z}_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \quad \left\{ \begin{array}{l} Z_L = 75\,\Omega \\ Z_0 = 50\,\Omega \\ \beta l = \frac{2\pi}{\lambda} \cdot 0.2\lambda \\ \quad = 72 \text{ degrees} \end{array} \right.$$

$$\tilde{Z}_1 = \frac{75 + j50 \cdot \tan(72°)}{50 + j75 \cdot \tan(72°)} \cdot Z_0 = \underline{35.2 - j8.6}$$

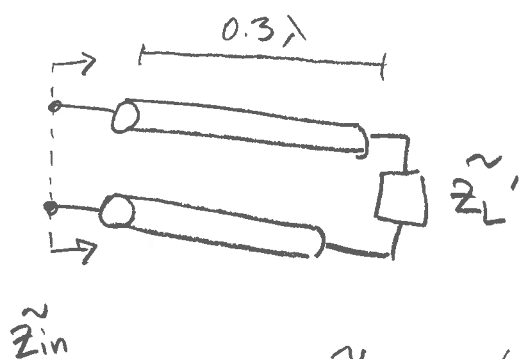
(b) Combine the two antennas impedances in parallel to find the total impedance at the Y-junction:



$$\tilde{Z}_L' = (\tilde{Z}_1 \parallel \tilde{Z}_1) = \frac{\tilde{Z}_1 \cdot \tilde{Z}_1}{\tilde{Z}_1 + \tilde{Z}_1} = \frac{1}{2}\tilde{Z}_1$$

$$\tilde{Z}_L' = (17.6 - j4.31)\,\Omega$$

(c) Transform this impedance all the way back to the feed point



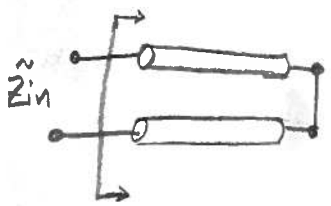
$$\tilde{Z}_{in} = \frac{\tilde{Z}_L' + jZ_0 \tan(\beta l)}{Z_0 + j\tilde{Z}_L' \tan(\beta l)} \cdot Z_0 \quad \left\{ \begin{array}{l} \tilde{Z}_L' = 17.6 - j4. \\ Z_0 = 50 \\ \beta l = \frac{2\pi}{\lambda} \cdot 0.3\lambda \\ \quad = 108° \end{array} \right.$$

$$\tilde{Z}_{in} = \frac{(17.6 - j4.31) + j50 \tan(108°)}{50 + j(17.6 - j4.31) \tan(108°)} \cdot Z_0$$

$$= ~~70.7~~$$

$$(107.6 - j56.7)\,\Omega$$

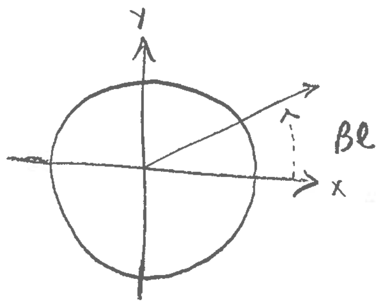
(2.24) Design a shorted transmission line to realize a reactance of $+j40\,\Omega$. Parameters: $f = 300$ MHz, $Z_0 = 50\,\Omega$, $u_p = 0.75c_0$



$$\tilde{Z}_{in} = j \cdot Z_0 \cdot \tan(\beta l) = +j40\,\Omega$$

$$\tan(\beta l) = \frac{40\,\Omega}{50\,\Omega} = 0.8$$

Draw this on a circle to work out the trig:



sketch shortest possible βl such that $\tan(\beta l) = 0.8$ and βl is still positive

$$\beta l = 38.66°$$

Use phase velocity to find β

$$u_p = \frac{\omega}{\beta} \xrightarrow[\text{for } \beta]{\text{solve}} \beta = \frac{\omega}{u_p} = \frac{300 \times 10^6 \times 2\pi \text{ rad/sec}}{0.75 \times 3 \times 10^8 \text{ m/s}}$$

$$\beta = 8.378 \text{ rad/m}$$

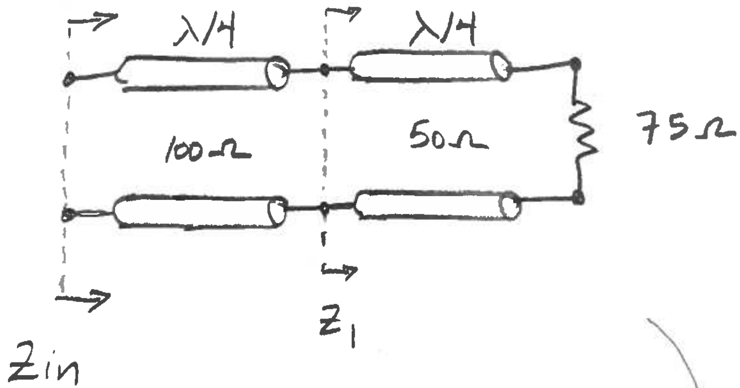
Put it all together to find l:

$$\beta l = 38.66° = 0.675 \text{ rad}$$

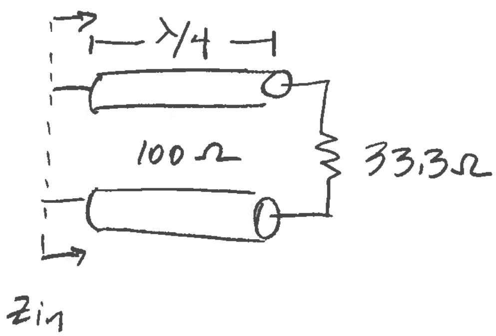
$$\therefore l = \frac{0.675 \text{ rad}}{\beta} = \frac{0.675 \text{ rad}}{8.378 \text{ rad/m}} \cong 0.08 \text{ m}$$

$$= 8 \text{ cm}$$

(2.27) Cascaded quarter wave transformers... use equation 2.77, twice.



$$Z_1 = \frac{(50\,\Omega)^2}{75\,\Omega} = 33.3\,\Omega$$



$$Z_{in} = \frac{(100\,\Omega)^2}{33.3\,\Omega} = 300\,\Omega$$

(2.35)

(a) $\tilde{Z}_L = 3Z_0$

$\tilde{z}_L = 3 + j0$

∴ $\tilde{\Gamma} \cong 0.5\angle 0°$

(b) $\tilde{Z}_L = (2-2j)Z_0$

$\tilde{z}_L = 2 - j2$

∴ $\tilde{\Gamma} \cong 0.628\angle 29.5°$

(c) $\tilde{Z}_L = -2jZ_0$

$\tilde{z}_L = -j2$

∴ $\tilde{\Gamma} \cong 1\angle -53°$

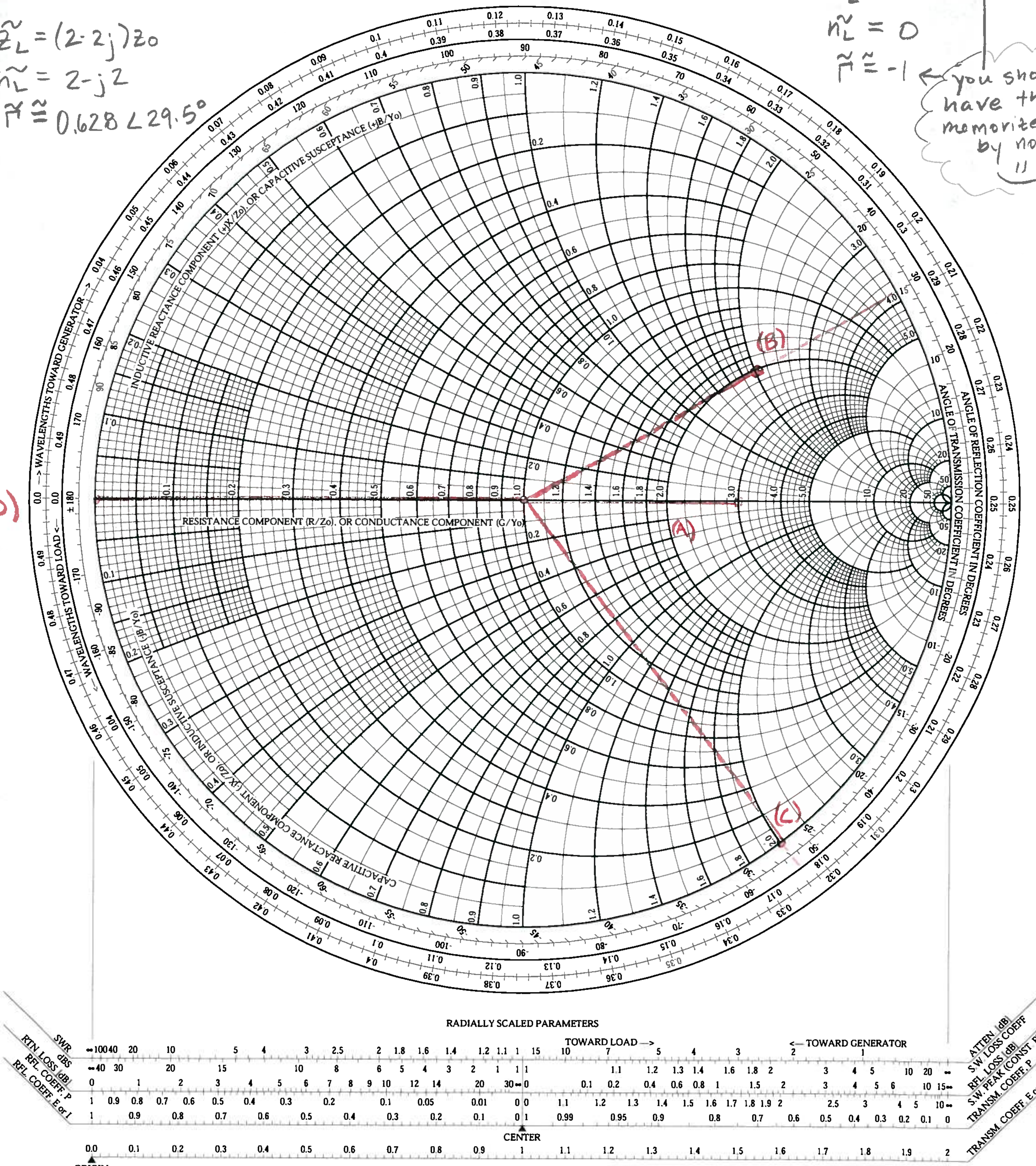
(d) $\tilde{Z}_L = 0$

$\tilde{z}_L = 0$

$\tilde{\Gamma} \cong -1$ ← you should have this memorized by now!!

The Complete Smith Chart

Black Magic Design



(2.36) (a) $\tilde{\Gamma} = 0.5$ $\quad \tilde{z}_L = 3 + j0$

(b) $\tilde{\Gamma} = 0.5 \angle 60^\circ$ $\quad \tilde{z}_L = 1.03 + j\,1.15$

(c) $\tilde{\Gamma} = -1$ $\quad \tilde{z}_L = 0$

(d) $\tilde{\Gamma} = 0.3 \angle -30^\circ$ $\quad \tilde{z}_L = 1.6 - j\,0.52$

(e) $\tilde{\Gamma} = 0$ $\quad z_L = 1 + 0j$

(f) $\tilde{\Gamma} = j$ $\quad z_L = 0 + j$

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