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ECE 311 - HW #5 $\{3.1,\ 3.5,\ 3.6,\ 3.33,\ 3.43,\ 4.9,\ 4.11\}$

(3.1) $\vec{A} =$ from $(1,-1,-3)$ to $(2,-1,0)$

$$\vec{A} = +1\,\hat{x} + 0\,\hat{y} + 3\,\hat{z}$$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{x}+3\hat{z}}{\sqrt{1^2+0^2+3^2}} = \frac{\hat{x}+3\hat{z}}{\sqrt{10}} = \frac{1}{\sqrt{10}}\hat{x} + \frac{3}{\sqrt{10}}\hat{z}$$

(3.5)

$$\vec{A} = \hat{x} + \hat{y}2 - \hat{z}3$$

$$\vec{B} = \hat{x}2 - \hat{y}4$$

$$\vec{C} = \hat{y}2 - \hat{z}4$$

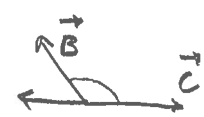
(a) $|A| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$ $\qquad \hat{A} = \frac{1}{\sqrt{14}}\hat{x} + \frac{2}{\sqrt{14}}\hat{y} - \frac{3}{\sqrt{14}}\hat{z}$

(b) Component of $\vec{B}$ along $\vec{C}$ $= \frac{\vec{B}\cdot\vec{C}}{|C|}\cdot\hat{C}$

$$= \frac{\langle 2,-4,0\rangle \cdot \langle 0,2,-4\rangle}{\sqrt{0^2+2^2+4^2}} \cdot \frac{\langle 0,2,-4\rangle}{\sqrt{0^2+2^2+4^2}}$$

$$= \frac{-8}{20}\langle 0,2,-4\rangle = \frac{-16}{20}\hat{y} + \frac{32}{20}\hat{z} = -0.8\hat{y} + 1.6\hat{z}$$

Back of book has $\frac{\vec{B}\cdot\vec{C}}{|C|} \cong -1.8$



$\frac{\vec{B}\cdot\vec{C}}{|C|}\hat{C} = -0.8\hat{y} + 1.6\hat{z}$

(c) $\theta_{AC} = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{C}}{|\vec{A}|\,|\vec{C}|}\right)$

$$= \cos^{-1}\left(\frac{\langle 1,2,-3\rangle \cdot \langle 0,2,-4\rangle}{\sqrt{14}\cdot\sqrt{20}}\right) = \cos^{-1}\left(\frac{16}{\sqrt{280}}\right) \cong 17.0^\circ$$

(d)

$$\vec{A}\times\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & 2 & -4 \end{vmatrix} = \hat{x}\begin{vmatrix} 2 & -3 \\ 2 & -4 \end{vmatrix} - \hat{y}\begin{vmatrix} 1 & -3 \\ 0 & -4 \end{vmatrix} + \hat{z}\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix}$$

$$= -2\hat{x} + 4\hat{y} + 2\hat{z}$$

(3.5) (e) $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = -\vec{B}\cdot(\vec{A}\times\vec{C})$

$= -\langle 2,-4,0\rangle \cdot \langle -2,4,2\rangle$

$= -(-4-16+0) = +20$

(f) $\vec{A}\times(\vec{B}\times\vec{C})$

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -4 & 0 \\ 0 & 2 & -4 \end{vmatrix} = 16\hat{x} + 8\hat{y} + 4\hat{z}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 16 & 8 & 4 \end{vmatrix} = \hat{x}\big(8-(-24)\big) + \hat{y}(-48-4) + \hat{z}(8-32)$$

$$= 32\hat{x} + -52\hat{y} - 24\hat{z}$$

(g) $$\hat{x}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 2 & -4 & 0 \end{vmatrix} = \hat{x}(0) + \hat{y}(0) + \hat{z}(-4) = -4\hat{z}$$

(h) $$(\vec{A}\times\hat{y})\cdot\hat{z} = \hat{z}\cdot\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & 1 & 0 \end{vmatrix} = \hat{z}\cdot\left(\hat{z}\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} + \underbrace{\hat{x}\,?? + \hat{y}\,??}_{\text{don't care}}\right)$$

$$= \hat{z}\cdot\hat{z}\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

3.6) $\vec{A} = \hat{x}2 - \hat{y} + \hat{z}3$

$\vec{B} = \hat{x}3 + 0\hat{y} - 2\hat{z}$

$$\vec{C} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}\cdot 9 \;; \quad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 2 & -1 & 3 \\ 3 & 0 & -2 \end{vmatrix} = 2\hat{x} + 13\hat{y} + 3\hat{z}$$

Alternately: $\vec{C} = \frac{\vec{B}\times\vec{A}}{|\vec{B}\times\vec{A}|}\cdot 9$

$$\vec{C} = \frac{\langle 2,13,3\rangle}{\sqrt{4+169+9}}\cdot 9 \cong 1.33\hat{x} + 8.67\hat{y} + 2.00\hat{z}$$

Alternately: $-1.33\hat{x} - 8.67\hat{y} - 2.00\hat{z}$

(3.33) $\nabla T = \hat{z}\, e^{-2z}$

By definition, $\nabla T = \hat{x}\dfrac{\partial T}{\partial x} + \hat{y}\dfrac{\partial T}{\partial y} + \hat{z}\dfrac{\partial T}{\partial z}$

So by inspection, $\dfrac{\partial T}{\partial z} = e^{-2z}$

anti-derivative

$$T(z) = -\frac{1}{2} e^{-2z} + C$$

Use the additional data, $T(z=0) = 10$, to establish C:

$$T(z=0) = -\frac{1}{2} e^{-2\cdot 0} + C = 10$$

$$-1/2 + C = 10$$

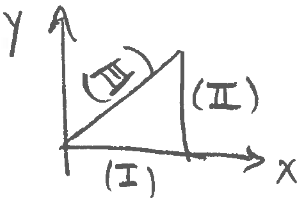
$$C = 10.5$$

Therefore, $T(z) = -\dfrac{1}{2} e^{-2z} + 10.5$

$$= 10 + \frac{1}{2}\left(1 - e^{-2z}\right)$$

(Textbook solution is in error.)

3.43) (a) Calculate $\oint \vec{E}\cdot d\vec{\ell}$ by splitting into 3 contours:



$$\oint \vec{E}\cdot d\vec{\ell} = \int_{(I)} \vec{E}\cdot d\vec{\ell} + \int_{(II)} \vec{E}\cdot d\vec{\ell} + \int_{(III)} \vec{E}\cdot d\vec{\ell}$$

First, path (I): $\displaystyle\int_{(I)} \vec{E}\cdot d\vec{\ell} = \int_0^1 \left[\hat{x}\,xy - \hat{y}(x^2+2y^2)\right]\cdot \hat{x}\,dx \Big|_{y=0}$

$= \displaystyle\int_0^1 xy\,dx \Big|_{y=0} = 0$ ← Since $y=0$ all along this path, theres no contribution.

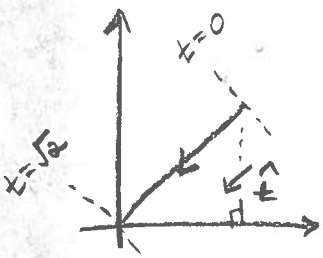
Now, path (II): $\displaystyle\int_{(II)} \vec{E}\cdot d\vec{\ell} = \int_0^1 \left[\hat{x}\,xy - \hat{y}(x^2+2y^2)\right]\cdot \hat{y}\,dy \Big|_{x=1}$

$= \displaystyle\int_0^1 -x^2 - 2y^2\,dy \Big|_{x=1} = \int_0^1 -1 - 2y^2\,dy$

$= \left[-y - \frac{2}{3}y^3\right]\Big|_{y=0}^{y=1} = \left[-1-\frac{2}{3}\right] - \left[0-0\right] = -5/3$

(3.43) (a) Continued...

Now do path (III). It's trickier ... reparameterize with respect to arclength, using the new variable, t.



Note that at $t=0$, $(x,y)=(1,1)$ and that at $t=\sqrt{2}$, $(x,y)=(0,0)$. Both x & y vary linearly with t... therefore:

$$x = \frac{\sqrt{2}-t}{\sqrt{2}} \qquad y = \frac{\sqrt{2}-t}{\sqrt{2}}$$ mapping from t to (x,y)

And the unit vector, $\hat{t} = -\hat{x}\frac{\sqrt{2}}{2} - \hat{y}\frac{\sqrt{2}}{2}$. Now we write:

$$\int_{(III)} \vec{E}\cdot d\vec{l} = \int_0^{\sqrt{2}} \vec{E}\cdot\hat{t}\,dt = \int_0^{\sqrt{2}} \left[\hat{x}(xy) - \hat{y}(x^2+2y^2)\right]\cdot\left[-\hat{x}\frac{\sqrt{2}}{2} - \hat{y}\frac{\sqrt{2}}{2}\right] dt$$

$$= \int_0^{\sqrt{2}} -\frac{\sqrt{2}}{2}(xy) + \frac{\sqrt{2}}{2}(x^2+2y^2)\,dt$$ substitute $y(t)$, $x(t)$

$$= \int_0^{\sqrt{2}} -\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}-t}{\sqrt{2}}\right)\left(\frac{\sqrt{2}-t}{\sqrt{2}}\right) + \frac{\sqrt{2}}{2}\left(\left\{\frac{\sqrt{2}-t}{\sqrt{2}}\right\}^2 + 2\left\{\frac{\sqrt{2}-t}{\sqrt{2}}\right\}^2\right) dt$$

$$= \int_0^{\sqrt{2}} \sqrt{2}\left(\frac{\sqrt{2}-t}{\sqrt{2}}\right)^2 dt = \frac{\sqrt{2}}{2}\int_0^{\sqrt{2}} (\sqrt{2}-t)^2\,dt$$

$$= \frac{\sqrt{2}}{2}\int_0^{\sqrt{2}} (2 - 2\sqrt{2}\,t + t^2)\,dt = \frac{\sqrt{2}}{2}\left[2t - \sqrt{2}\,t^2 + \frac{1}{3}t^3\right]_{t=0}^{t=\sqrt{2}}$$

$$= \frac{\sqrt{2}}{2}\left[\cancel{2\sqrt{2}} - \cancel{2\sqrt{2}} + \frac{1}{3}\sqrt{2}^{\,3}\right]$$

$$= \frac{\sqrt{2}}{2}\cdot 2\sqrt{2}\cdot\frac{1}{3} = +\frac{2}{3}$$ (whew!)

$$\therefore \oint \vec{E}\cdot d\vec{l} = \int_{(I)} \vec{E}\cdot d\vec{l} + \int_{(II)} \vec{E}\cdot d\vec{l} + \int_{(III)} \vec{E}\cdot d\vec{l}$$

$$= 0 + -5/3 + 2/3 = -1$$

3.43)(b) By Stoke's theorem, this should be the same as (a)...

$$\vec{E} = \hat{x}\, xy - \hat{y}\,(x^2 + 2yz)$$

$$\nabla \times \vec{E} = \hat{x}\left(\underbrace{\frac{\partial E_z}{\partial y}}_{\to 0} - \underbrace{\frac{\partial E_y}{\partial z}}_{\to 0}\right) + \hat{y}\left(\underbrace{\frac{\partial E_x}{\partial z}}_{\to 0} - \underbrace{\frac{\partial E_z}{\partial x}}_{\to 0}\right) + \hat{z}\left(\underbrace{\frac{\partial E_y}{\partial x}}_{\neq 0!} - \underbrace{\frac{\partial E_x}{\partial y}}_{\neq 0!}\right)$$

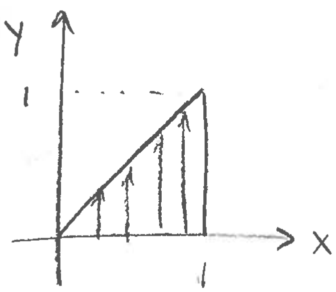
$$= \hat{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= \hat{z}\left(-2x - x\right) = -3x \cdot \hat{z}$$

$$\iint \nabla \times \vec{E} \cdot d\vec{s} = \int_0^1 \int_0^x (-3x\,\hat{z}) \cdot \hat{z}\, dy\, dx$$

y ranges from 0 to x

x ranges from 0 to 1



$$= \int_0^1 \int_0^x (-3x)\, dy\, dx$$

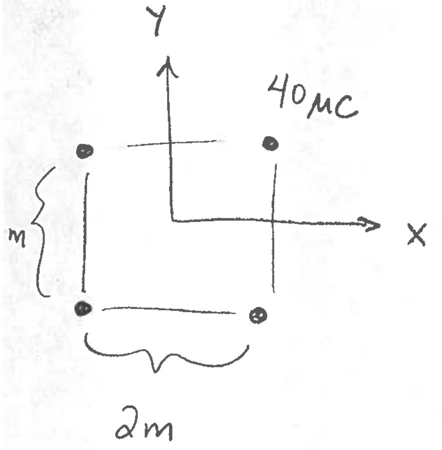
$$= \int_0^1 \left(-3xy \Big|_{y=0}^{y=x}\right) dx$$

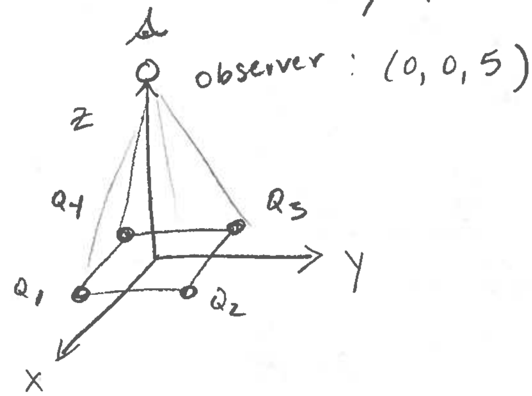
$$= \int_0^1 -3x^2 \; dx$$

$$= (-x^3)\Big|_{x=0}^{x=1} = -1$$

Same result as part (a), as expected.

(4.9) Sketch the problem. Place the origin at the center of the square, in the x-y plane:





According to coulombs law:

$$\vec{E}_i = \frac{Q_i \cdot (\vec{r} - \vec{r}_i)}{4\pi\varepsilon_0 \cdot |\vec{r} - \vec{r}_i|^3} \quad ; \quad \vec{E}_{total} = \sum_{i=1}^{4} (\vec{E}_i)$$

The distance to each charge is the same, can be factored out:

$$\vec{E}_{total} = \frac{40\text{ nC}}{4\pi\varepsilon_0 \left(\sqrt{1^2+1^2+5^2}\right)^3} \cdot \sum_{i=1}^{4} (\vec{r} - \vec{r}_i)$$

$$= \frac{40\text{ nC}}{4\pi\varepsilon_0 (27)^{3/2}\text{ m}^3} \cdot \left\{ \begin{array}{ll} \langle 0,0,5\rangle - \langle +1,-1,0\rangle + & \leftarrow Q_1 \\ \langle 0,0,5\rangle - \langle +1,+1,0\rangle + & \leftarrow Q_2 \\ \langle 0,0,5\rangle - \langle -1,+1,0\rangle + & \leftarrow Q_3 \\ \langle 0,0,5\rangle - \langle -1,-1,0\rangle & \leftarrow Q_4 \end{array} \right\}$$

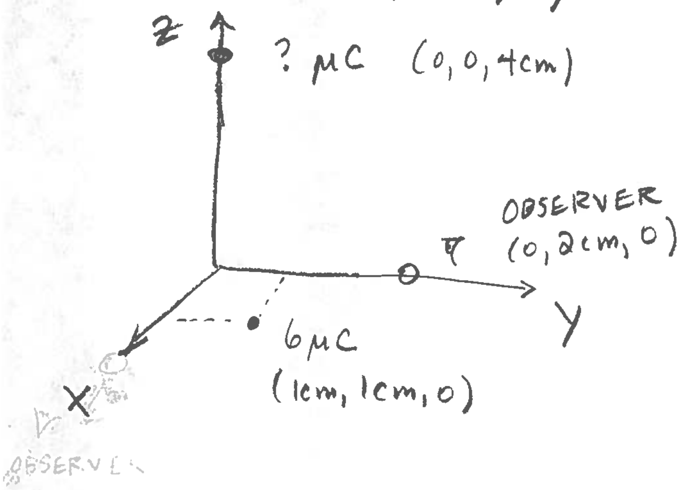
all these cancel!

$$\vec{E}_{tot} = \frac{40\text{ nC}}{4\pi\varepsilon_0 (27)^{3/2}\text{ m}^3} \cdot \langle 0, 0, 20\text{ m}\rangle$$

$$= \frac{(40\text{ nC})(20\text{m})\,\hat{z}}{4\pi \cdot (8.85\times10^{-12}\text{ F/m})\,(27)^{3/2}\text{ m}^3} \cong \hat{z}\, 5.1273 \times 10^4 \text{ V/m}$$

$$\cong 51.2\text{ kV/m} \cdot \hat{z}$$

(4.11) Place 6 μC at (1cm, 1cm, 0). How much charge would need to be placed at (0,0,4cm) such that $\vec{E}$ at (0,2cm,0) has no y-component?



Write the total field at the observation point... all units in cm.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{6\mu C \cdot (\langle 0,2,0\rangle - \langle 1,1,0\rangle)}{|\langle 0,2,0\rangle - \langle 1,1,0\rangle|^3} + \frac{Q_2 \cdot (\langle 0,2,0\rangle - \langle 0,0,4\rangle)}{|\langle 0,2,0\rangle - \langle 0,0,4\rangle|^3} \right\}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{6\mu C \langle -1,1,0\rangle}{(\sqrt{2})^3} + \frac{Q_2 \langle 0,2,-4\rangle}{(\sqrt{20})^3} \right\}$$

Desire $E_y = 0$ ... dot $\vec{E}$ with $\hat{y}$ and set to zero...

$$\hat{y}\cdot\vec{E} = \frac{1}{4\pi\varepsilon_0}\left\{ \frac{6\mu C}{(\sqrt{2})^3} + \frac{2Q_2}{(\sqrt{20})^3} \right\} = 0$$

Now solve for Q_2, to find what value yields $E_y = 0$...

$$\frac{6\mu C}{(\sqrt{2})^3} + \frac{2Q_2}{(\sqrt{20})^3} = 0$$

$$\frac{2Q_2}{(\sqrt{20})^3} = \frac{-6\mu C}{(\sqrt{2})^3} \quad \therefore \quad Q_2 = \frac{-6\mu C}{(\sqrt{2})^3} \cdot \frac{(\sqrt{20})^3}{2}$$

$$Q_2 = -94.87\ \mu C$$