Ryan Chilton
ECE 311 - HW #6

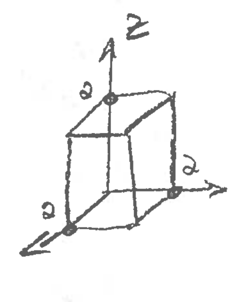
(4.20) (a) $\rho_v = \nabla \cdot \vec{D} = \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$

$= \frac{\partial}{\partial x}(2x+2y) + \frac{\partial}{\partial y}(3x-2y)$

$= 2 - 2 = 0 \ C/m^3$

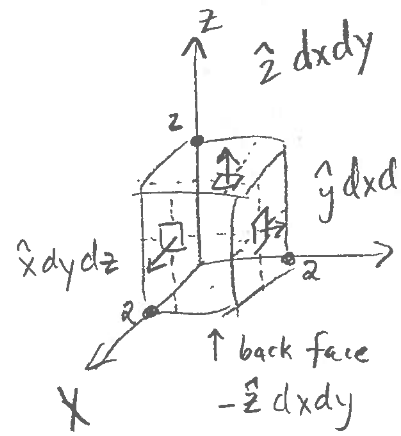
(b) $Q_{total} = \iiint \rho_v \, dv = \int_0^2 \int_0^2 \int_0^2 0 \, dx\,dy\,dz$

$= 0 \ C$



(c) $Q_{total} = \oint\!\!\oint_S \vec{D} \cdot d\vec{s}$

Break this into $\underline{\underline{6}}$ surface integrals:



$Q = \int_0^2 \int_0^2 \vec{D} \cdot \hat{z}\,dx\,dy \,\Big|_{z=2m} +$

$- \int_0^2 \int_0^2 \vec{D} \cdot \hat{z}\,dx\,dy \,\Big|_{z=0m} +$

$\int_0^2 \int_0^2 \vec{D} \cdot \hat{y}\,dx\,dz \,\Big|_{y=2m} - \int_0^2 \int_0^2 \vec{D} \cdot \hat{y}\,dx\,dz \,\Big|_{y=0m}$

$+ \int_0^2 \int_0^2 \vec{D} \cdot \hat{x}\,dy\,dz \,\Big|_{x=2m} - \int_0^2 \int_0^2 \vec{D} \cdot \hat{x}\,dy\,dz \,\Big|_{x=0m}$

Here, $\vec{D}$ has no $\hat{z}$ component, so the first 2 integrals vanish.

$Q = \int_0^2 \int_0^2 (3x-2y)\,dx\,dz \,\Big|_{y=2m} - \int_0^2 \int_0^2 (3x-2y)\,dx\,dz \,\Big|_{y=0}$

$+ \int_0^2 \int_0^2 (2x+2y)\,dy\,dz \,\Big|_{x=2m} - \int_0^2 \int_0^2 (2x+2y)\,dy\,dz \,\Big|_{x=0}$

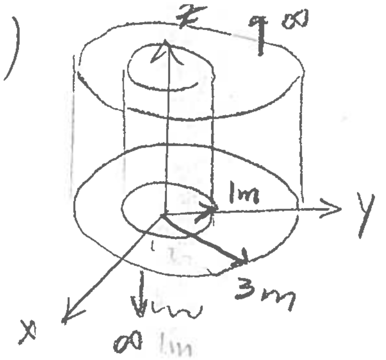
$Q = \int_0^2 \int_0^2 (3x-4)\,dx\,dz - \int_0^2 \int_0^2 3x \,dx\,dz$

$+ \int_0^2 \int_0^2 (4+2y)\,dy\,dz - \int_0^2 \int_0^2 2y\,dy\,dz$ $\quad = -4 - 12 + 24 - 8 = 0!$

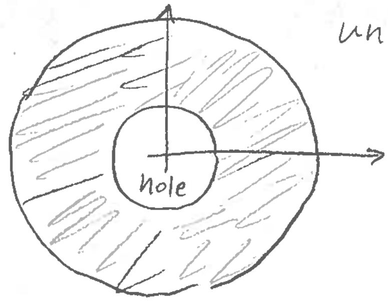
$= 2 \cdot \left(\frac{3}{2}x^2 - 4x\right)\Big|_0^2 - 2 \cdot \left(\frac{3}{2}x^2\right)\Big|_0^2 + 2\left(4y + y^2\right)\Big|_0^2 - 2\left(y^2\right)\Big|_0^2$

$= -4 + 12 + 24 - 8 = 0$

(4.25)



Top view:



uniform charge density ρ_{v0}

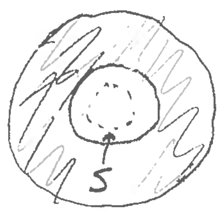
By symmetry considerations, $\vec{D}$ only has an $\hat{r}$ component, and is invariant under z translation and x,y rotation (that is, no dependence on ϕ or z). We write:

$$\vec{D} = \hat{r}\, D_r(r)$$

Theres 3 cases to consider as Gaussian Surfaces, S ...

FIRST

$r < 1m$



$$\oiint \vec{D}\cdot d\vec{s} = Q\text{ inside}$$ only cylinder wall contributes

$$\int_0^h \int_0^{2\pi} \hat{r} D_r \cdot \underbrace{\hat{r}(r\,d\phi\,dz)}_{d\vec{s}} = 0$$ (no charge!)

jacobian

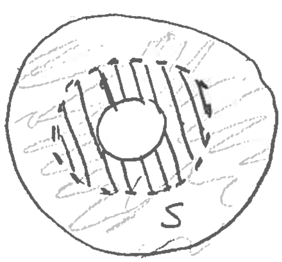
$$\int_0^h \int_0^{2\pi} D_r \cdot r\,d\phi\,dz = 0$$

$$D_r \cdot 2\pi r h = 0$$

$$D_r = 0 \qquad r < 1m$$

SECOND:

$1m < r < 3m$



$$\oiint \vec{D}\cdot d\vec{s} = Q\text{inside}$$

$$\int_0^h \int_0^{2\pi} D_r \cdot r\,d\phi\,dz = \rho_{v_0}\left[\pi r^2 h - \pi(1m)^2 h\right]$$

(uniform charge density)

(Volume of hatched shell)

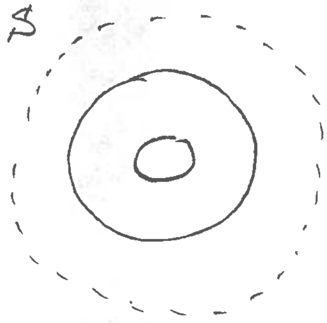
$$D_r \cdot 2\pi r h = \rho_{v0}\, \pi h\,(r^2 - 1m^2)$$

$$D_r = \frac{\rho_{v0}\,(r^2 - 1m^2)}{2r} \qquad 1m < r < 3m$$

C/m2

THIRD

$r > 3\,m$



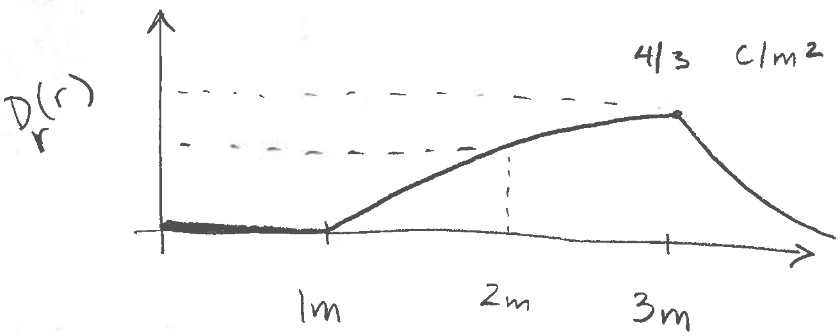
$$\oiint \vec{D}\cdot d\vec{s} = Q_{inside}$$

$$\int_0^h \int_0^{2\pi} D_r \cdot r\, d\phi\, dz = \rho_{v_0}\left[\pi (3m)^2 h - \pi (1m)^2 h\right]$$

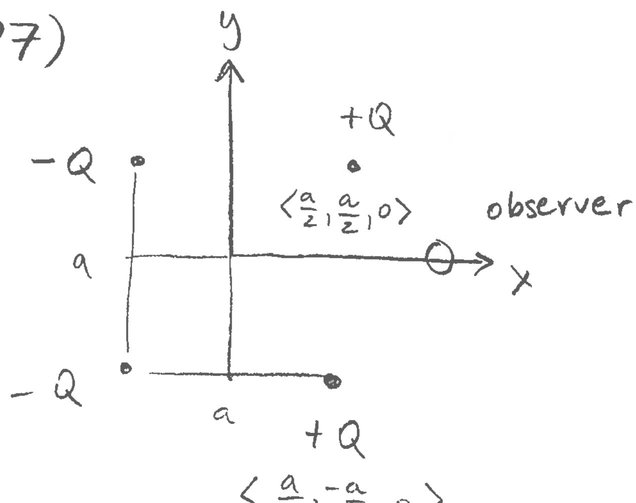
$$2\pi r h \cdot D_r = (\rho_{v_0} \cdot 8\pi h) \text{ Coulombs}$$

$$D_r = \frac{\rho_{v_0}\cdot 4}{r}\ C/m^2 \quad r > 3m$$

Heres a sketch:



4.27)



Define a point on the x-axis:

$$\vec{R} = \langle x, 0, 0 \rangle$$

Sum over discrete charges, adding up voltage contributions:

$$V_{total} = \sum_{i=1}^{4} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{|\vec{R}-\vec{R}_i|}$$

(a)

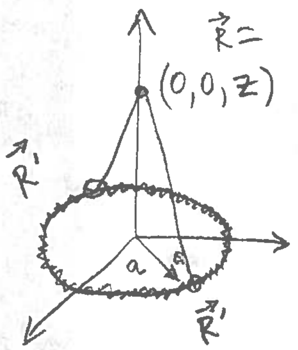
$$V_{total}(x) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{\sqrt{(x-\frac{a}{2})^2+(\frac{a}{2})^2}} + \frac{Q}{\sqrt{(x-\frac{a}{2})^2+(\frac{a}{2})^2}} + \frac{-Q}{\sqrt{(x+\frac{a}{2})^2+(\frac{a}{2})^2}} + \frac{-Q}{\sqrt{(x+\frac{a}{2})^2+(\frac{a}{2})^2}} \right)$$

(b) Evaluate at $x = \frac{a}{2}$

$$V_{total}\left(\frac{a}{2}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(a/2)} + \frac{Q}{(a/2)} + \frac{-Q}{\sqrt{a^2+a^2/4}} + \frac{-Q}{\sqrt{a^2+a^2/4}} \right)$$

$$= \frac{1}{4\pi\varepsilon_0} Q \left(\frac{2}{a} + \frac{2}{a} + \frac{-2}{a\sqrt{5}} + \frac{-2}{a\sqrt{5}} \right) = \frac{Q}{\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{5}}\right)$$

(4.29) Circular ring of charge with radius a, observed at a point $(0,0,z)$:



Note as we walk around the ring, the distance from the differential piece of charge to the observer doesn't change! (it's always $\sqrt{z^2+a^2}$)

Use: $V = \frac{1}{4\pi\varepsilon_0} \int_{L'} \frac{\rho_\ell}{|\vec{R}-\vec{R}'|} dL'$ over the circular path L'...

$$V = \frac{1}{4\pi\varepsilon_0} \int_{\phi=0}^{\phi=2\pi} \frac{\rho_\ell}{\sqrt{z^2+a^2}} \cdot r\,d\phi$$

(walking around the circle)

$\rho_\ell / |\vec{R}-\vec{R}'|$

jacobian

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \int_{\phi=0}^{\phi=2\pi} \frac{\rho_\ell}{\sqrt{z^2+a^2}} \cdot a\,d\phi$$

integration path is constant ρ ($r=a$)

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_\ell \cdot a}{\sqrt{z^2+a^2}} \left(\int_0^{2\pi} d\phi\right) = \frac{1}{2\varepsilon_0} \cdot \frac{\rho_\ell \cdot a}{\sqrt{z^2+a^2}}$$

(b) Compute the -gradient of V to find $\vec{E}$. Note that V is only a function of z...

$$\vec{E} = -\nabla V = \hat{z} \cdot -\frac{\partial}{\partial z}\left[V(z)\right]$$

$$= -\hat{z} \cdot \frac{\rho_L \cdot a}{2 \cdot \varepsilon_0} \cdot \frac{\partial}{\partial z}\left[\frac{1}{\sqrt{z^2+a^2}}\right]$$

$$= +\hat{z}\,\frac{\rho_\ell \cdot a}{2\varepsilon_0} \cdot \frac{z}{(z^2+a^2)^{3/2}}$$

$$\frac{\partial}{\partial z}\left[\frac{1}{\sqrt{z^2+a^2}}\right] = \frac{\partial}{\partial z}(z^2+a^2)^{-1/2}$$

$$= -\frac{1}{2}(z^2+a^2)^{-3/2} \cdot \frac{\partial}{\partial z}(z^2+a^2)$$

CHAI RU + POWE RUL

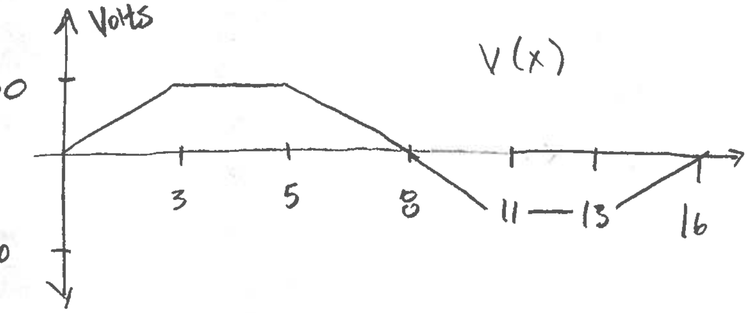
$$= -\frac{1}{2}(z^2+a^2)^{-3/2} \cdot (2z)$$

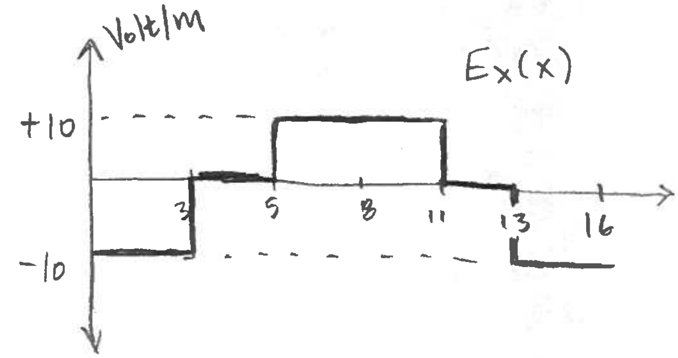
$$= -z(z^2+a^2)^{-3/2}$$

(4.33) Given the potential graph, plot the electric field.

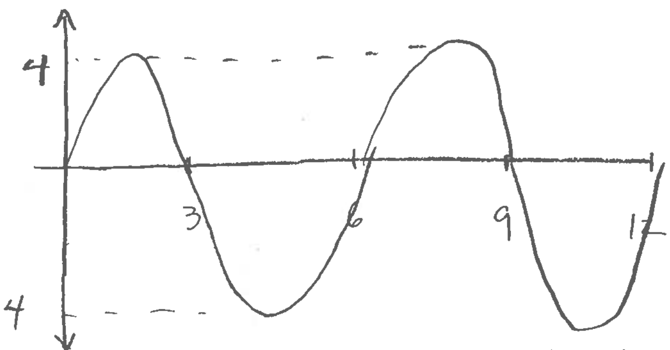
$$\vec{E} = -\nabla\left[V(x)\right] = \hat{x}\cdot\underbrace{-\frac{\partial}{\partial x}V(x)}_{\text{"}E_x\text{"}}$$

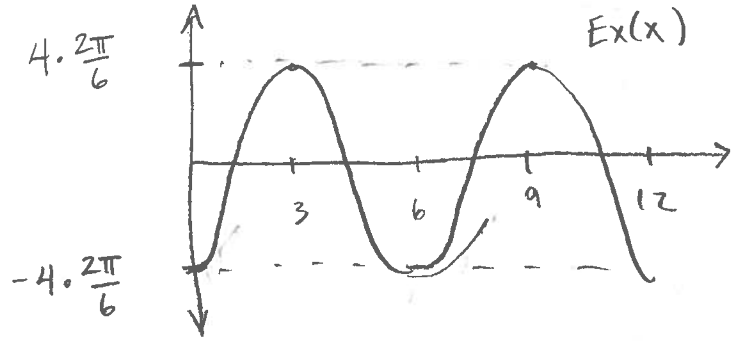
a)





b)

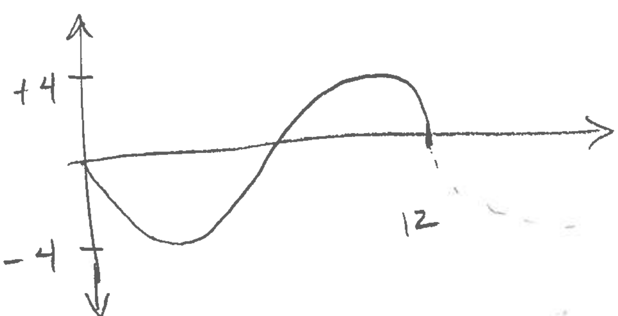


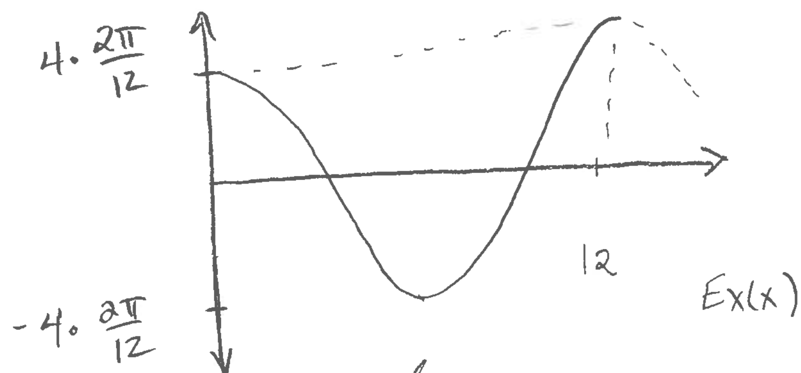


period = 6m, freq = $^{1\,cycle}/_{6m}$ = $^{2\pi\,rad}/_{6m}$

sinusoid - differentiates to cosine, scaled by frequency

c)





Now the period is 12m, and we flipped the sign...

lousy artwork...