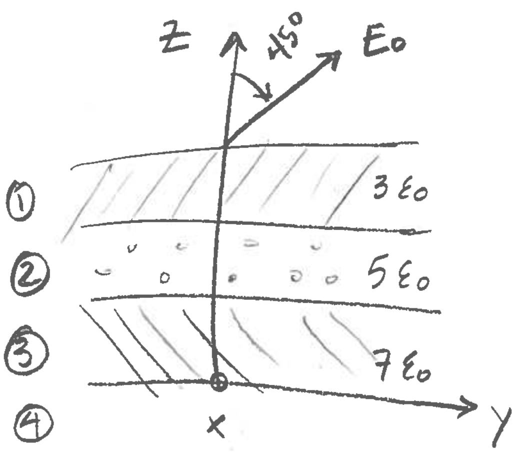
Ryan Chilton
ECE 311 HW # 8

(4.47)



By tangential field continuity, every layer will have the same E_y component:

$$E_y = E_0 \cdot \sin(45°) = \frac{\sqrt{2}}{2} E_0$$

At dielectric/dielectric interfaces, there is no charge density, so normal flux is also continuous at all interfaces:

$$D_z = \varepsilon_0 E_0 \cdot \cos(45°) = \varepsilon_0 E_0 \cdot \frac{\sqrt{2}}{2}$$

Each layer has different ε, so they have different E_z:

$$E_{z,1} = \frac{\varepsilon_0 E_0 \cdot \frac{\sqrt{2}}{2}}{3\varepsilon_0} = E_0 \frac{\sqrt{2}}{6}$$

$$E_{z,2} = E_0 \cdot \sqrt{2}/10 \qquad E_{z,3} = E_0 \cdot \frac{\sqrt{2}}{14}$$

$$E_{z,4} = E_0 \cdot \frac{\sqrt{2}}{2}$$

Compute the angle with respect to the z-axis in each layer

$$\theta_1 = \operatorname{atan}\left(\frac{E_{y,1}}{E_{z,1}}\right) = \operatorname{atan}\left(\frac{\frac{\sqrt{2}}{2} E_0}{\frac{\sqrt{2}}{6} E_0}\right) = \operatorname{atan}(3) \cong 71.6°$$

$$\theta_2 = \operatorname{atan}(5) = 78.7°$$

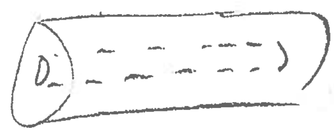
$$\theta_3 = \operatorname{atan}(7) = 81.9°$$

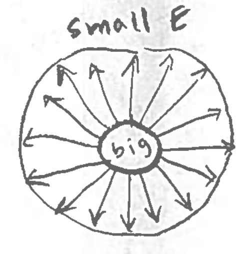
$$\theta_4 = \operatorname{atan}(1) = 45°$$

49) (a) According to Example 4-12,

$$\vec{E} = -\hat{r}\,\frac{Q}{2\pi\varepsilon r L}$$

if $a < r < b$





So $|\vec{E}|$ is inversely proportional to r. That means that $|\vec{E}|$ will be maximized when r is minimized. Pick $r = a$ - $|\vec{E}|$ is maximum at the inner conductor.

(b) Mica has a dielectric strength of 200 MV/m. If $|\vec{E}|$ exceeds this value, the mica will breakdown/conduct. Breakdown is most likely to occur at the inner conductor

$$E = \frac{Q}{2\pi\varepsilon r L}$$

set $E = 200$ MV/m and $r = 1$cm

$$200\,\frac{MV}{m} = \frac{Q}{(2\pi\varepsilon)\cdot(1cm)\cdot L} \longrightarrow (200\,MV/m)\cdot(1cm) = \frac{Q}{2\pi\varepsilon L}$$

"breakdown equation"

According to (4.115), the voltage of the coax line is given by:

$$V = \frac{Q}{2\pi\varepsilon L}\ln(b/a)$$

The ratio, $\frac{Q}{2\pi\varepsilon L}$, is known from the breakdown equation:

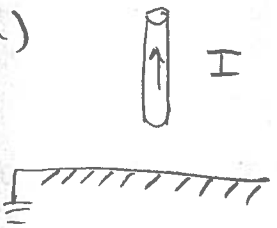
$$V = (200\,MV/m)(1cm)\cdot \ln(b/a)$$

$$V = (2MV)\cdot \ln(2cm/1cm)$$

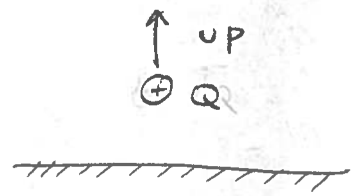
$$V = 1.39\ MV$$

(4.5 7) Moving charges (currents) and image theory:

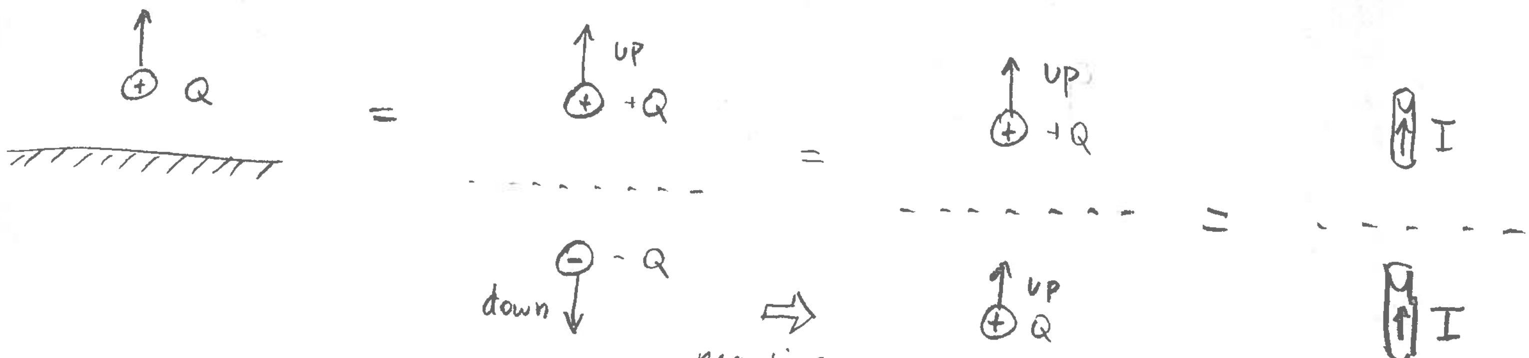
(a)



→ This current indicates positive charge, moving upwards. →



Now apply image theory: mirror the charge and negate its polarity:



negative charge going down is same as positive charge going up

b)

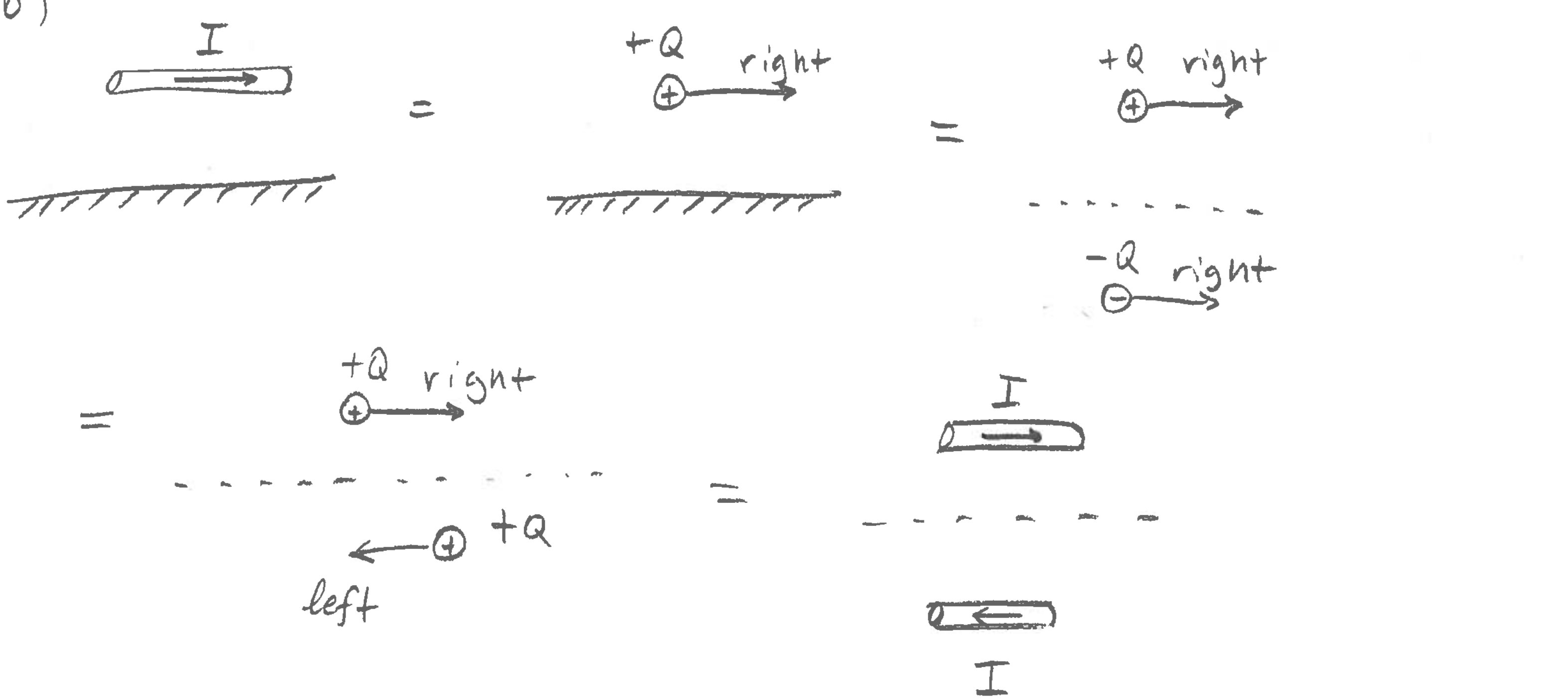


Image theory: tangential current flips direction, normal current stays same direction

(5.1) $\vec{v} = \hat{x} \cdot 8 \times 10^6$ m/s

$\vec{B} = (\hat{x}4 + \hat{z}3)$ T

$q = -1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ kg

$\vec{a} = \vec{F}/m = (q/m) \cdot \vec{v} \times \vec{B}$

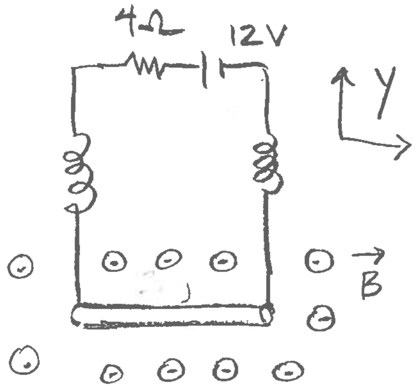
$\vec{a} = \left(\frac{-1.6 \times 10^{-19} C}{9.1 \times 10^{-31} kg}\right)(8 \times 10^6 m/s)(3T)(\hat{x} \times -\hat{z})$

$\vec{a} = -4.22 \times 10^{18} \cdot \left(\frac{\cancel{C}}{kg} \cdot \frac{\cancel{m}}{\cancel{s}} \cdot \frac{N \cdot \cancel{s}}{\cancel{C} \cdot \cancel{m}}\right)(+\hat{y})$

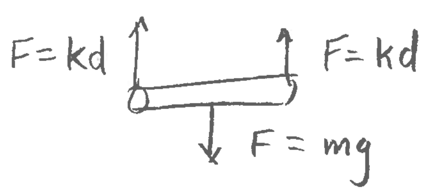
$\vec{a} = -\hat{y}\, 4.22 \times 10^{18} \cdot N/kg$ $\quad (N = kg \cdot \frac{m}{s^2})$

$\vec{a} = -\hat{y}\, 4.22 \times 10^{18}\ m/s^2$

(5.3)



First, use the rest position to find the stretching constant of the springs. Free body diagram:



displacement, d= 0.2c[m]

gravity, $g = 9.8\ m/s^2$

mass, m= 20g

Equilibrium: net force = 0

$2 \cdot k \cdot d = mg$

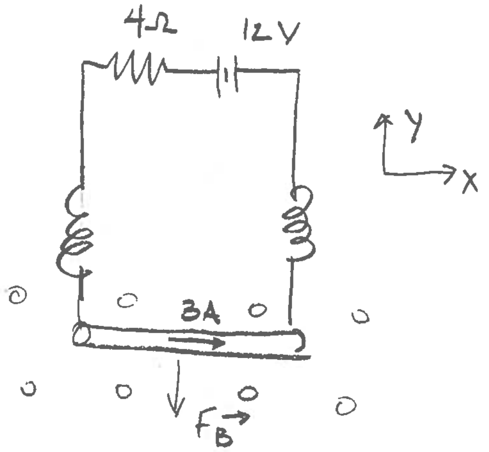
$2 \cdot k\,(0.2\,cm) = (20g)(9.8\ m/s^2)$

$2 \cdot k\,(2 \times 10^{-3} m) = 0.196$ N

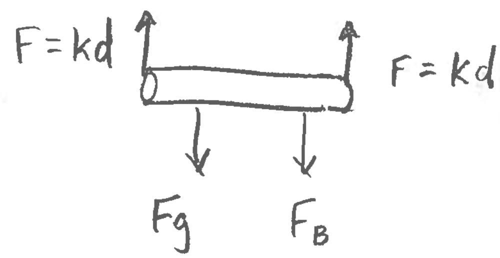
$k = 49$ N/m

(Continued other side)

When the circuit is driven, a current flows in the wire and $\vec{B}$ pushes this wire with an additional force, $\vec{F}_B = \int d\vec{L} \times \vec{B} \cdot I$



Draw another free body diagram:



Equilibrium - forces sum to zero; solve for magnetic force

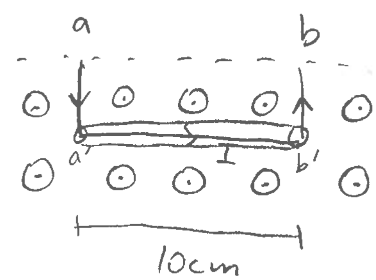
$$2kd = g \cdot m + F_B$$

$$2(49\ \text{N/m})(0.7\ \text{cm}) - (9.8\ \text{m/s}^2)(20\text{g}) = F_B$$

$$0.49\ \text{N} = F_B$$

Now: I, F_B and the path $\int d\vec{L}$ are known. Find $\vec{B}$:

$$\vec{F}_B = I\left(\int_a^b d\vec{L}\right) \times \vec{B}$$



note: force on the section from a to a' will be equal & opposite the force on the section b'-b

$$-\hat{y} \cdot 0.49\ \text{N} = (3\text{A})\underbrace{\left(\int_{a'}^{b'} d\vec{L}\right)}_{\text{this is } +10\text{cm}\ \hat{x}} \times (B_0\ \hat{z})$$

$$-\hat{y} \cdot 0.49\ \text{N} = (3\text{A})(+10\text{cm})(B_0)(\underbrace{\hat{x} \times \hat{z}}_{-\hat{y}})$$

$$0.49\ \text{N} = (3\text{A})(10\ \text{cm})(B_0) \longrightarrow B_0 = \frac{0.49\ \text{N}}{3(\text{C/s}) \cdot (10\text{cm})} = 1.63\ \text{T}$$