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ECE 311 - HW #9

(5.9)

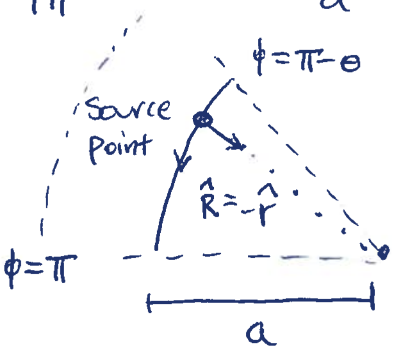
Biot-Savart Law:

$$d\vec{H} = \frac{I}{4\pi}\,\frac{d\vec{L}\times\hat{R}}{R^2}$$

Immediately, we notice that current segments ① ② and ③ will produce no contribution to $\vec{H}$ at o, because $d\vec{L}$ is parallel to $\hat{R}$ (thus $d\vec{L}\times\hat{R} = 0$)

Only the circular arcs contribute:

$$\frac{I}{4\pi}\int_{\pi-\theta}^{\pi}\frac{\hat{\phi}\times(-\hat{r})}{a^2}\,a\,d\phi \;+\; \frac{I}{4\pi}\int_{\pi-\theta}^{\pi}\frac{-\hat{\phi}\times(-\hat{r})}{b^2}\cdot b\,d\phi = \vec{H}$$



INNER ~~ACR~~ ARC

OUTER ARC

Note: $\hat{\phi}\times(-\hat{r}) = +\hat{z}$

$-\hat{\phi}\times(-\hat{r}) = -\hat{z}$

$$\vec{H} = \hat{z}\cdot\frac{I}{4\pi}\int_{\pi-\theta}^{\pi}\frac{1}{a}\,d\phi - \hat{z}\,\frac{I}{4\pi}\int_{\pi-\theta}^{\pi}\frac{1}{b}\,d\phi$$

$$= \hat{z}\cdot\frac{I}{4\pi a}\underbrace{\int_{\pi-\theta}^{\pi}d\phi}_{\theta} - \hat{z}\,\frac{I}{4\pi b}\underbrace{\int_{\pi-\theta}^{\pi}d\phi}_{\theta}$$

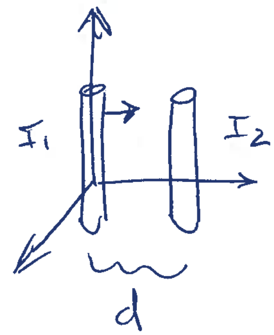
$$\vec{H} = \hat{z}\cdot\frac{I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \hat{z}\cdot\frac{I\theta}{4\pi}\left(\frac{b}{ba}-\frac{a}{ba}\right) = \hat{z}\,\frac{I\theta}{4\pi}\left(\frac{b-a}{ab}\right)$$

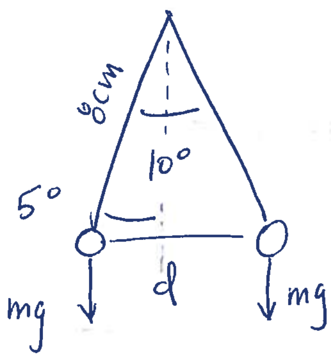
(5.16) Recall class example, force per unit length between to conductors. (In book, on pp. 218)

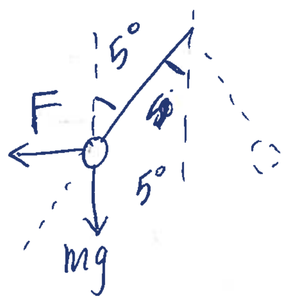
$$\vec{F}_1' = \hat{y}\,\frac{\mu_0 I_1 I_2}{2\pi d} \quad (N/m)$$

(attractive ~~force~~ force for parallel I, repulsive force for anti parallel I)

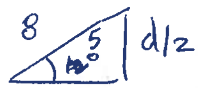


Draw cross-section sketch: gravity wants to pull this thing straight down. Must be some repulsive force pushing them apart. Thus, currents must be opposite.



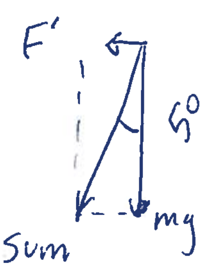


Note: $d = 2\cdot 8\cdot \sin(5^\circ)$



$d = 1.3945$ cm

Furthermore, for this thing to be at equilibrium, the vector sum of the gravity force and repulsive force must point along the direction of the tension force in the string...



therefore: $\tan(5^\circ) = \dfrac{F'}{mg}$ (key step!)

$$F' = mg\tan(5^\circ)$$

$$\frac{\mu_0 \cdot I^2}{2\pi\cdot d} = (1.2\ g/cm)\cdot(9.82\ m/s^2)\cdot\tan(5^\circ)$$

$$I^2 = \sqrt{\left(0.12\ \frac{kg}{m}\right)\left(9.8\ \frac{m}{s^2}\right)\cdot(2\pi\cdot 1.39\times10^{-2}\ m)\,/\,\mu_0}$$

where $\mu_0 = \cancel{8.85\times10}$ $4\pi\times10^{-7}$ H/m

$H = \dfrac{m^2\cdot kg}{s^2\cdot A^2}$ $H/m = \dfrac{m\cdot kg}{s^2\cdot A^2}$

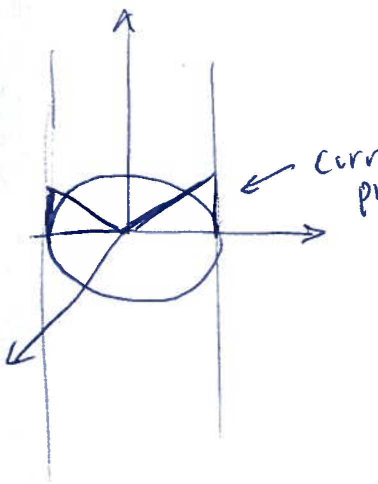
$\cancel{I = 359\ \mu A}$

$$I^2 = \frac{0.12 \cdot 9.8 \cdot 2\pi \cdot 1.39 \times 10^{-2}}{4\pi \times 10^{-7}} \cdot \frac{\cancel{kg}}{\cancel{m}} \cdot \frac{\cancel{m}}{\cancel{s^2}} \cdot m \cdot \frac{s^2 \cdot A^2}{m \cdot \cancel{kg}}$$

$$I^2 = 8.17 \times 10^4 \ A^2$$

$$I = 285 \ A \quad (\text{wow!})$$

(5.21) $\vec{J} = J_0 \cdot \hat{z} \cdot \frac{1}{r}$



By symmetry considerations,

$$\vec{H} = \hat{\phi} \cdot H_\phi(r) \qquad \{\text{as always}\}$$

Apply amperes law inside the cylinder:

$$\oint \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{s}$$

$$\int_0^{2\pi} \hat{\phi}\, H_\phi(r) \cdot r \hat{\phi}\, d\phi = \int_0^{2\pi} \int_0^{r} J_0 \hat{z} \frac{1}{x} \cdot \hat{z} \cdot x \, d\phi \, dx$$

$$2\pi r \cdot H_\phi(r) = 2\pi J_0 \cdot \int_0^r dx$$

$$2\pi r \cdot H_\phi = 2\pi J_0 \cdot r$$

$$H_\phi = J_0$$

$$\vec{H} = \hat{\phi} J_0 \qquad (r < a)$$

Now try outside the loop:

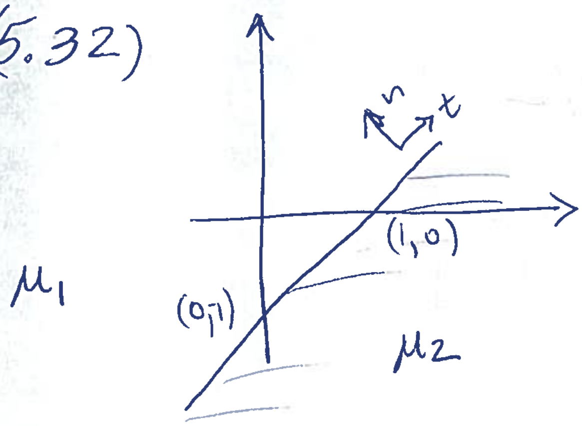
$$2\pi r \cdot H_\phi = \int_0^{2\pi} \int_0^{a} \left(J_0 \cdot \hat{z} \cdot \frac{1}{r}\right) \cdot \hat{z}\, r \, d\phi \, dr$$

$$2\pi r \cdot H_\phi = \int_0^{2\pi} \int_0^{a} J_0 \, d\phi \, dr$$

$$2\pi r \cdot H_\phi = 2\pi a \cdot J_0$$

$$H_\phi = \frac{J_0 \cdot a}{r} \qquad (r > a)$$

(5.32)



Arbitrary oriented surface

$$\vec{B}_1 = \hat{x}2 + \hat{y}3 \ \mathrm{T}$$

Find tangent and normal vectors:

$$\hat{t} = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \nearrow$$

$$\hat{n} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle \nwarrow$$

Normal flux continuity tells us:

$$\hat{n} \cdot \vec{B}_1 = \hat{n} \cdot \vec{B}_2$$

$$\left(-\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y}\right) \cdot (\hat{x}2 + \hat{y}3)\mathrm{T} = \left(-\frac{\sqrt{2}}{2}\hat{x} + \frac{\sqrt{2}}{2}\hat{y}\right) \cdot \vec{B}_2$$

$$\left(-\sqrt{2} + \frac{3\sqrt{2}}{2}\right)\mathrm{T} = -\frac{\sqrt{2}}{2}B_{2,x} + \frac{\sqrt{2}}{2}B_{2,y}$$ (one linear constraint on $B_{2,x}$ and $B_{2,y}$)

$$\frac{\sqrt{2}}{2}\mathrm{T} = -\frac{\sqrt{2}}{2}B_{2,x} + \frac{\sqrt{2}}{2}B_{2,y}$$

Tangential field continuity tells us:

$$\hat{t} \cdot \vec{H}_1 = \hat{t} \cdot \vec{H}_2$$

$$\sqrt{2}/2\, H_{1,x} + \frac{\sqrt{2}}{2}H_{1,y} = \frac{\sqrt{2}}{2}H_{2,x} + \frac{\sqrt{2}}{2}H_{2,y}$$ (now use $H = \frac{B}{\mu}$)

$$\frac{\sqrt{2}}{2}\frac{B_{1,x}}{\mu_1} + \frac{\sqrt{2}}{2}\frac{B_{1,y}}{\mu_1} = \frac{\sqrt{2}}{2}\frac{B_{2,x}}{\mu_2} + \frac{\sqrt{2}}{2}\frac{B_{2,y}}{\mu_2}$$ (now use $\mu_1 = 5\mu_2$)

$$\frac{\sqrt{2}}{10}B_{1,x} + \frac{\sqrt{2}}{10}B_{1,y} = \frac{\sqrt{2}}{2}B_{2,x} + \frac{\sqrt{2}}{2}B_{2,y}$$ (now, $B_{1,x} = 2\mathrm{T}$, $B_{1,y} = 3\mathrm{T}$)

$$\left(\frac{\sqrt{2}}{5} + \frac{3\sqrt{2}}{10}\right)\mathrm{T} = \frac{\sqrt{2}}{2}B_{2,x} + \frac{\sqrt{2}}{2}B_{2,y}$$ (another linear constraint)

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} & +\frac{\sqrt{2}}{2} \\ +\frac{\sqrt{2}}{2} & +\frac{\sqrt{2}}{2} \end{bmatrix} \begin{bmatrix} B_{2,x} \\ B_{2,y} \end{bmatrix} = \begin{bmatrix} \sqrt{2}/2 \\ \frac{\sqrt{2}}{5} + \frac{3\sqrt{2}}{10} \end{bmatrix}$$

$$\begin{bmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} B_{2,x} \\ B_{2,y} \end{bmatrix} = \begin{bmatrix} 1/2 \\ \frac{1}{5} + \frac{3}{10} = 1/2 \end{bmatrix}$$

$$\begin{bmatrix} B_{2,x} \\ B_{2,y} \end{bmatrix} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}^{-1} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$B_{2,x} = 0 \text{ T}$

$B_{2,y} = 1 \text{ T}$